

Working at the Intersection of Research and Practice: A Perspective on the Study and Improvement of Mathematics Lessons

Yoshinori Shimizu
University of Tsukuba, Japan
<yshimizu@human.tsukuba.ac.jp>

Given the distinctive characteristics of Japanese mathematics lessons found by the international studies and the subsequent focus of the attention to Lesson Study in mathematics education community, the author discusses the relationships between the scientific studies and improvement of mathematics teaching and learning from an insider's perspective. Reflecting on how mathematics educators have been struggling with studying the complex phenomena called "lesson" in the Japanese context, it is argued that orchestrating the scientific goal of building theories and the goal of improving teaching and learning of mathematics is the key to a synergy between research and the more practical knowledge of the craft of teaching.

The findings of large-scale international studies of classroom practices in mathematics include aspects of instruction as identified among participating countries while instruction in Japan is seemingly unique (Hiebert, et al., 2003; Stigler & Hiebert, 1999). Japanese mathematics teachers, for example, appeared to spend more time on the same task in one lesson than their counterparts in other countries (Hiebert et al., 2003, p.46), maybe because they have students work on a challenging problem and discuss alternative solutions to it. Also, experienced teachers in Japan tend to highlight and summarise the main points at particular phases of their lesson to have their students reflect on what they have learned (Shimizu, 2006). These striking characteristics can be regarded as indicating some indispensable elements of mathematics classroom instruction that are valued and emphasized by Japanese teachers.

While much has been documented and analysed about mathematics classrooms in Japan, after the publication of *The Teaching Gap* (Stigler & Hiebert, 1999), in particular, Lesson Study became one of the notable topics in the mathematics education research community. Lesson Study, 'jugyo kenkyu' in Japanese, is a common element in Japanese approach to improving teaching and learning in classroom whereby a group of teachers collaborate to study the subject matter, students' thinking and learning in the classroom, and how classroom instruction can be improved (Fernandez & Yoshida, 2004; Lewis & Tsuchida, 1998; Shimizu, 2002). Two major functions of Lesson Study are as a way of doing research with a hypothesis in the form of conducting lesson and as a place for presenting and discussing new findings based on classroom practice (Hirabayashi, 2002). These functions are related to the rationale for conducting Lesson Study as an opportunity of professional development for teachers to study the effectiveness of mathematics teaching and learning in their own classrooms. Also, practical knowledge related to the improvement of classroom instruction is accumulated as 'research' findings tested against the classroom practices of many teachers for many years.

Given the tradition of Lesson Study, that has a history of more than a century, Japanese mathematics educators have often been challenged by teachers who have been engaged in Lesson Study whether their research results would or would not be usefully applicable to the improvement of teaching and learning in the classroom (Sekiguchi, 1994). The expectations by teachers for mathematics education research are high because they have

their own problems to be resolved in their own context. If a professor who was invited to Lesson Study as an outside expert could not meet the standards for their ‘needs’, he or she might never be invited to the school as an advisor. In such cases, suggestions and advice by the researcher could be ‘too theoretical’ or ‘with little relevance to our school.’ How can a researcher productively and collaboratively work with teachers who have an access to the accumulation of practical knowledge tested against the classroom practices of many teachers for many years? How can research influence on classroom practices in such contexts?

An essential characteristic of the field of mathematics education is that its questions and concerns are deeply tied to matters related to the teaching and learning of mathematics (Silver & Herbst, 2007). Research in mathematics education has not only scientific goal of building theories but also the goal of improving of teaching and learning of mathematics. Then, examining the connections between ‘scientific’ study of a lesson and Lesson Study can shed light on a long-standing issue of the separation between research and practice in the education community, in general, and mathematics education community, in particular.

Given the distinctive characteristics of Japanese mathematics lessons found by the international studies and the subsequent focus of attention to Lesson Study in mathematics education research community, the author discusses the relationships between the scientific studies and improvement of mathematics teaching and learning from an insider’s perspective. By reflecting on how mathematics educators have been struggling with studying complex phenomena called ‘lesson’ (Jugyo) in Japanese context, it is argued that orchestrating the scientific goal of building theories and the goal of improving of teaching and learning of mathematics is the key to a synergy between research and the more practical knowledge of the craft of teaching. .

Research as Socially and Culturally Situated

International Comparative Studies on Classroom Practices

Research in mathematics education that crosses national boundaries provides new insights into the development and improvement of the teaching and learning of mathematics (Shimizu & Kaur, 2013). In the course of discussing the characteristics of teaching and learning in classrooms by cross-national comparisons, researchers have gained more explicit understanding of their own implicit theories about how teachers teach and how children learn mathematics in their local contexts as well as what is going on in school mathematics in other countries (Stigler, Gallimore, & Hiebert, 2000). This is the key driving force to conduct international comparative studies.

The TIMSS 1995 Video Study of mathematics teachers’ practices, a video component of the Third International Mathematics and Science Study, was the first attempt to collect and analyse videotapes from the classrooms of national probability samples of teacher at work (Stigler & Hiebert, 1999). Focusing on the actions of teachers, it has provided a rich source of information regarding what goes on inside eighth-grade mathematics classes in Germany, Japan and the United States with certain contrasts among three countries. One of the sharp contrasts between the lessons in Japan and those in the other two countries relates to how lessons were structured and delivered by the teacher. The structure of Japanese lessons was characterized as ‘structured problem solving’, while a focus was on procedures in the characterisations of lessons in the other two countries. The following sequence of

five activities was described as the ‘Japanese pattern’: reviewing the previous lesson; presenting the problems for the day; students working individually or in groups; discussing solution methods; and highlighting and summarizing the main point.

The ‘Japanese pattern’ seems to naturally fit within the teacher’s planning of mathematics lessons. Japanese teachers, in elementary and junior high schools, in particular, often organise an entire mathematics lesson around the multiple solutions to a single problem in a whole-class instructional mode (Shimizu, 1999a). This organization is particularly useful when a new concept or a new procedure is going to be introduced during the initial phase of a teaching unit. Even during the middle or final phases of the teaching unit, teachers often organize lessons by posing a few problems with a focus on the various solutions students come up with. Also, the characterisation of Japanese lessons as ‘structured problem solving’ seems to be consistent with what teachers typically discuss in the post-lesson discussion in Lesson Study (Takahashi, 2008).

Characterization of the practices of a nation’s or a culture’s mathematics classrooms with a single lesson pattern was, however, problematised by the results of the Learner’s Perspective Study (Clarke, Mesiti, O’Keefe, Jablonka, Mok & Shimizu, 2007). The analysis suggested that, in particular, the process of mathematics teaching and learning in Japanese classrooms could not be adequately represented by a single lesson pattern by, at least, the following two reasons. First, lesson pattern differs considerably within one teaching unit, which can be a topic or a series of topics, depending on the teacher’s intentions throughout the sequence of lessons. Second, elements in the pattern themselves can have different meanings and functions in the sequence of multiple lessons. Needless to say, it is an important aspect of teacher’s work not only to implement a single lesson but also to weave multiple lessons that can stretch out over several days, or even a few weeks, into a coherent body of the unit. It would not be possible for us to capture the dynamic nature of activities in teaching and learning process if each lesson was analysed as isolated.

An alternative approach was proposed to the international comparisons of lessons by the researchers in LPS team. That is, a postulated ‘lesson event’ was regarded to serve as the basis for comparisons of classroom practice internationally. In LPS, an analytical approach was taken to explore the form and functions of the particular lesson events such as ‘between desk instruction’, ‘students at the front’, and ‘highlighting and summarizing the main point’ (Clarke, Emanuelsson, Jablonka & Mok, 2006).

In particular, the form and functions of the particular lesson event ‘highlighting and summarizing the main point’, or ‘Matome’ in Japanese, were analyzed in eighth-grade ‘well-taught’ mathematics classrooms in Australia, Germany, Hong Kong, Japan, Mainland China (Shanghai), and the USA (Shimizu, 2006). For the Japanese teachers, the event ‘Matome’ appeared to have the following principal functions: (i) highlighting and summarizing the main point, (ii) promote students’ reflection on what they have done, (iii) setting the context for introducing a new mathematical concept or term based on the previous experiences, and (iv) making connections between the current topic and previous one. For the teachers to be successful in maintaining these functions, the goals of lesson should be very clear to themselves, activities in the lesson as a whole need to be coherent, and students need to be involved deeply in the process of teaching and learning. The results suggest that clear goals of the lesson, a coherence of activities in the entire lesson, active students’ involvement into the lesson, are all to be noted for the quality instruction in Japanese classrooms.

Mathematics Education Research as Socially and Culturally Situated

In the Japanese contexts, the activity of Lesson Study includes careful planning and implementing the research lesson as a core of the whole activity, followed by post-lesson discussion and reflection by participants. In the discourse of teachers in planning, implementing, and reflecting on lessons, particular pedagogical terms are often shared and used in the contexts of examining classroom instruction (Shimizu, 1999). Through the participation in lesson study, beginning teachers learn these terms together with values attached to them. These terms reflect what Japanese teachers value in planning and implementing lesson within Japanese culture. 'Matome' as a category of 'lesson event', as was mentioned in the previous section, is one of such pedagogical terms often used by teachers. "Hatsumon" is another example of shared term that means asking a key question to provoke and facilitate students' thinking at a particular point of the lesson. The teacher may ask a question for probing students' understanding of the topic at the beginning of the lesson or for facilitating students' thinking on the specific aspect of the problem.

"Yamaba", on the other hand, means a highlight or climax of a lesson. Japanese teachers think that any lesson should include at least one "Yamaba" and the term often appears in planning a lesson and the post-lesson discussion in Lesson Study. This climax usually appears as a highlight during the whole-class discussion. The point here is that all the activities, or some variations of them, constitute a coherent system called a lesson that hopefully include a climax. Further, among Japanese teachers, a lesson is often regarded as a drama, which has a beginning, leads to a climax, and then invites a conclusion. The idea of "ki-sho-ten-ketsu" which was originated in the Chinese poem, is often referred by Japanese teachers in their planning and implementation of a lesson. It is suggested that Japanese lessons have a particular structure or a flow, moving from the beginning ("ki", a starting point) toward the end ("ketsu", summary of the whole story).

As teaching is a cultural activity and is socially and culturally situated, research is also socially and culturally situated. Sekiguchi (in press) argues that there are at least three major sources of Japanese mathematics education research; lesson study tradition, influence from Western countries, and existence of national syllabus. Then, in addition to focus on pedagogical terms shared and used in Lesson Study by teachers with values and beliefs on lessons, it would be productive for researchers to take into account the following contexts to have better understanding of classroom practices. First, in Japan we have national curriculum standards, which have been revised roughly every 10 years. In order to examine the trends and issues in most areas of mathematics research in Japan, we cannot neglect their connections with the goals and emphases described in the national curriculum standards. Second, the mathematics education community in Japan has a long tradition of lesson study by teachers as practical research methodologies in the form of action research. Researchers and teachers work closely within the community with local theories of students' learning in their perspective. Then, to grasp the ongoing research agendas, we also need to pay careful attention to their accumulated findings with respect to teaching materials and ways of teaching and learning in each research area. Third, developments of mathematics education research in Japan have been influenced by Western educational theories in various areas of inquiry, while educational activities themselves are rooted in East Asian cultural tradition.

Lesson Study as Embedded in Cultural Contexts

The ultimate goal of any study of classroom practice is to improve teaching for the purpose of enhancing students' learning, even if its major focus would be on, for example, constructing a theoretical model of teaching or comparing teaching methods among countries. For this goal, various approaches and methodologies can be adapted. The TIMSS Video Study was a breakthrough as a scientific exploration into the classroom, showing the feasibility of applying videotape methodology in a wide-scale national and international survey of classroom instructional practice. It has provided a rich source of information regarding what goes on inside eighth-grade mathematics classes in the three countries. Also, objective observational measures of classroom instruction were developed to serve as valid quantitative indicators, at a national level, of teaching practices in the three countries.

There are opportunities for Japanese teachers to learn with and from their experienced colleagues to pursue an excellent lesson with a focus on students' thinking in classrooms. "Lesson study" is an approach to develop and maintain quality mathematics instruction through a particular form of activity (Fernandez & Yoshida, 2004; Shimizu, 2002). Valuing students' thinking as necessary elements to be incorporated into the development of a lesson is a key to the approach taken by Japanese teachers.

Another type of approach to improve teaching practice has traditionally been taken by Japanese teachers in a practical way. In Japanese schools, workshops of particular style, "Jugyo Kenkyu-kai" ("lesson study meeting"), are regularly held at each school level or at the other levels. This opportunity has a strong impact on teacher development (Shimizu, 1999). These workshops include an actual lesson (a "research lesson") observed by many teachers as well as an extended discussion after the lesson (Lewis & Tsuchida, 1998). Teachers exchange ideas about the lesson they have just observed focusing on the task on which students worked, the content taught, students' responses to the problem, and the teacher's roles. Experienced teachers or mathematics educators are often invited to comment on the development of the lesson observed, interpretations of the topic taught, and how the lesson could be improved. By observing and discussing an actual lesson, aspects of lessons are examined and explored.

The Origin and the Development of Lesson Study

Although Fernandez and Yoshida (2004) mention that the origin of lesson study can be traced back to the early 1890s, it seems to have appeared earlier. At the beginning of the modern era, the Japanese government established normal schools, where teachers set the goals of the lesson, prepared experimental lessons, and conducted those lessons in actual classrooms while other teachers were observing them. In the late 1890s, teachers at elementary schools affiliated to the normal schools started to study lessons by observing and examining them critically. Makinae (2010) argues that the origin of Japanese lesson study was influenced during the late 1880's by U.S. books for educators that introduced new approaches to teach. He points out that a book by Sheldon (1862) describes methods to learn about new teaching approaches, called 'criticism lesson' and 'model lesson'. This may be the beginning of Japanese lesson study. In fact, Inagaki (1995) argues that 'criticism lesson' was already practiced among elementary schools affiliated to the normal schools in Japan as early as the late 1890s. Teacher conferences utilising criticism lessons

were conducted by local school districts in the early 1900s. Some of these conferences were already called ‘lesson study conferences’, or *jugyo-kenkyu-kai* in Japanese (Makinae, 2010). In this sense, lesson study has a history of more than a century.

In the early stages of development of Japanese lesson study, ‘criticism lesson’ (Sheldon, 1862) included a particular function of studying lessons, carefully examining the effectiveness of teaching, and publicly discussing ways to improve teaching and learning. The term ‘research lesson’, or *kenkyu-jugyo*, might come from this particular function of lesson study with its major focus on producing a new idea, or testing a hypothesis in the form of an operationalized teaching method or teaching materials. On the other hand, ‘model lesson’ (Sheldon, 1862) included another function of studying lessons; demonstrating or showcasing exemplary lessons, or presenting new approaches for teaching. For this purpose, the lesson should be carefully planned and based on research conducted by a teacher or a group of teachers. Participants can observe and discuss actual lessons with a hypothesis, instead of simply reading papers or hand-outs that describe the results of the study. The two different functions of lesson study – ‘criticism lesson’ and ‘model lesson’ – can be the original model of a variety of lesson study practiced around the country.

Despite the long history of lesson study in their own country, Japanese mathematics educators and researchers in other areas have not been much interested in studying lesson study itself until recently. After the publication of *The Teaching Gap* (Stigler & Hiebert, 1999), and of a Japanese translation of the book (Minato, 2002), Japanese educators, who often deeply involved in lesson study, have “found” the importance of this particular cultural activity.

Today, lesson study takes place in various institutions and contexts (Lewis & Tsuchida, 1998; Shimizu, 2002). Pre-service teacher training programs at universities and colleges, for example, include lesson study as a crucial and challenging part in the final week of student teaching practice, which usually lasts three or four weeks. In-service teachers also have opportunities to participate, held within their school (*konai-kenshu*), outside their school but in the same school district or city, at the level of prefecture, and even at the national level for several objectives. Teachers at public schools may just participate in lesson study in their school to develop their teaching skills, since the school is their working place. Other teachers may play the major roles in planning and conducting research lesson, for testing critically their hypothesis in the use of particular method for teaching mathematics. Teachers at university-affiliated schools that have a mission to developing a new approach to teaching, often open their lesson study meeting for demonstrating an approach or new teaching materials they have developed. Thus, we can still see two major functions of lesson study that seems to have arisen from the original form of it.

The Role of Outside Expert in Lesson Study

In lesson study, an outside expert is often invited as an advisor who facilitates the post-lesson discussion and/or makes comments on the possible improvement of lesson from a broader viewpoint (Fernandez and Yoshida, 2004; Shimizu, 2008). The expert may be an experienced teacher, a supervisor at local board of education, a principal of a different school, or a professor from the nearby university. In some cases, not only inviting an expert as a commentator in the post-lesson discussion, the group of teachers may meet with him/her several times prior to conducting the research lesson to discuss issues such as

reshaping the objective of the lesson, clarifying the rationale of a particular task to be presented in the classroom, a range of anticipatory student responses to the task, and so on. In this context, an outside expert can be a collaborator who shares responsibility for the quality of a lesson with the teachers, not just an outside authority which directs the team of teachers.

As for the university professor invited as an outside expert, he or she is expected as a researcher to provide new visions on curriculum reform and teaching practices, trends and issues in local and national educational policies, and also some concrete suggestions for improving daily classroom practices, as well as commenting on what was observed in research lesson. Given the tradition of lesson study, mathematics educators have often been challenged by school teachers who deeply engaged in lesson study whether “research results” provided by researchers are useful for improving classroom practices.

There is another role of an outside expert. Namely, the expert can be as a collaborator and a contributor who joins the process of planning, implementing, and reflecting on a research lesson together with the group of teachers. In this case, an outside expert can be seen as a part of the community of practice, not an authority that has come from outside the school.

Studying and Improving of Teaching and Learning of Mathematics

Working with and Learning from Teachers

There has been a concern in mathematics education community, at least in Japan, with the relevance and usefulness of the results of research. Also, it is often argued that there is a long-standing issue of the separation between research and practice in education community, in general, and mathematics education community, in particular. Given the tradition of lesson study, it is very important for Japanese mathematics educators to work with and to learn from teachers. The relationship between research and practice may be seen differently in other countries.

Among five crucial relationships in research in mathematics education that he identified as important, Bishop (1992) lists the relationship between the teacher and the researcher as a particularly significant one. He characterizes three theoretical traditions, pedagogue, empirical scientist, and scholastic philosopher, and each tradition has the goal of enquiry, role of evidence, and role of theory in different ways. If the goal of study is direct improvement of teaching, and role of theory is accumulated and sharable wisdom of expert teachers, the study is in the pedagogue tradition. The evidence presented is usually highly selective and exemplary here. He noted that in both empirical scientist and scholastic philosopher traditions, the roles of teacher and researcher are incompatible. He wrote:

The teacher is the practitioner whose practice, it is felt, needs to be informed by the research of the researcher. So, we have a clear hierarchy involved, with the researcher informing the teacher, but not necessary vice versa (p.717).

Bishop (1992) noted that the analysis and study of mathematics teaching from both these perspectives can make the teacher an object – not a subject – in the research. The role of outside expert who is invited to Lesson Study as an advisor can be considered in light of this hierarchy. The expert can facilitate the post-lesson discussion and/or makes comments on the possible improvement of lesson from a broader viewpoint. Or, being involved in

Lesson Study can invite the consequence for the researcher being just an outside authority coming into the classroom to direct the group of teachers and develop theories for them, if the researcher is not aware of their role and does not understand the significance of working with and learning from teachers.

There is a possibility that the gap may become larger between the efforts in mathematics education research and the problems tackled by the teachers. Lerman (1990) noted that to make separation between those who practice, and those who develop theories for the practitioners, is not an adequate characterization of the business of good teaching. Also, Wiliam & Lester (2008) wrote as follows.

We promote a renewal of a sense of purpose for our research activity that seems to be disappearing, namely, a concern for making real, positive, lasting changes in what goes on in classrooms. We suggest that such changes will occur only when we become more aware of and concerned with sharing of meaning across researchers and practitioners. (p. 38)

One of the major characteristics of Lesson Study is that, as the historical development illustrates, the approach was initiated by a group of teachers to improve teaching and learning in classrooms. The problem tackled by teachers has rooted in the reality of the school and the classroom. Research questions can be posed in responding to problems derived from teachers' works. In working with teachers and learning from teachers in Lesson Study, mathematics educators can have an opportunity for identifying implicit wisdom and accommodating craft knowledge to scholarly knowledge. Ellerton & Clement (1994) raised the issues to be confronted by the international mathematics education research community and noted the need to demonstrate a greater respect for the wisdom of practice deriving from the classroom knowledge and the action-oriented theories of practicing teachers of mathematics in different countries around the world. As Lesson Study has become a focus of attention in countries including Australia, Malaysia, and the United States, for example, we have more chances to learn from the voice of teachers.

Studying One Lesson Intensively

Lesson study usually is conducted in a few classrooms and then its results have very limited validity in the beginning. If those results are shared among other teachers, and replicated in many other classrooms in different schools, they could increasingly obtain higher validity and relevance. Therefore, a larger community of teachers who wish to learn from other teachers are key for the success of lesson study with certain generalizability. The key is a particular focus on aspects of developing a lesson. Participating in lesson study provides opportunities for teachers to learn shared values of teaching mathematics as a school subject, with and from experienced colleagues. Such values are related to teachers' views on a 'good' lesson and an 'excellent' teacher.

Ruthven & Goodchild (2008) discuss the significant of craft knowledge, the professional knowledge used by teachers in their day-to-day classroom teaching; action-oriented knowledge which is not generally made explicit by teachers, which they may indeed find difficult to articulate, or which they may even be unaware of using. Orchestrating scientific goal of building theories and the goal of improving of teaching and learning of mathematics is the key to a synergy between research and the more practical knowledge of the craft of teaching. Lewis et al. (2006) argues that development of a descriptive knowledge base, explication of an innovation's mechanism, and iterative cycles of improvement research. By studying a good lesson continuously, practical knowledge

tested against the classroom practices of many teachers for many years is accumulated. Here research can inform practice by providing a tool for explaining in broader views or providing teachers categories for describing the meaning of their vocabulary.

To develop better understandings of educational activities in local contexts, researchers need to consider the underlying values and beliefs shared by that local community (Shimizu & Williams, 2012). It should be noted, for instance, that valuing students' thinking as necessary elements to be incorporated into the development of a lesson is key to the approach taken by Japanese teachers (Shimizu, 2009). Describing anticipated students' responses, is, amongst other activities, key to lesson planning because the whole-class discussion depends on the solution methods the students actually come up with. Having a very clear sense of the ways students are likely to think about and solve a problem prior to the start of a lesson makes it easier for teachers to know what to look for when they are observing students work on the problem.

Mary Hesse (1980), pointed out that any theory is value-laden, and that an awareness of the value behind a theory is crucial, in social science, in particular. In any science, criteria for theory choices contain value judgments, but those values tend to be filtered out as theories are developed. We need to be conscious that educational theories are value-laden and that those values derived in part from our experience with educational practices in a broader sense. Japanese mathematics educators who are engaged in Lesson Study are interested in "good" or "successful" classroom practice. The tendency is derived from their experiences with Lesson Study that has the goal of building theories and the goal of improving teaching and learning of mathematics at the same time.

Concluding Remarks

For more than a decade, educators and researchers in the field of mathematics education have been interested in Lesson Study as a promising source of ideas for improving education. For a Japanese mathematics educator who has been deeply involved in lesson study for more than two decades, this 'movement' has provided an opportunity for reflecting on how Lesson Study as a cultural activity works as a system embedded in the entire society as well as local community of teachers with shared values and beliefs. It is argued that orchestrating the scientific goal of building theories and the goal of improving of teaching and learning of mathematics is the key to a synergy between research and the more practical knowledge of the craft of teaching. International comparative studies have started to recognize the need to focus more on existing diverse voices and perspectives among members of the local communities. As the globalization and internationalization of research activities has continued to expand, the field of mathematics education research has clearly shown the diversification of perspectives on teaching and learning in classrooms embedded in local contexts. As the classroom practices are socially and culturally situated, and shared values and beliefs by teachers are key for continuous development and of the quality teaching, research in mathematics education is socially and culturally situated. Continuous working with, and learning from, teachers raises the issues and shapes the research questions originated in the efforts of improvement of teaching and learning mathematics in the classroom.

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